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ABSTRACT

It is noted that helping children learn how to solve problems has become a major goal for school mathematics. However, many children are not very successful at deciding what operation is appropriate for solving a given problem, and teachers often have difficulty describing or explaining how to identify the key features of a particular problem that enable them to decide which operation is appropriate. The focus of this report is on two complementary approaches to helping children learn how to decide when to use a problem-solving operation. The results of these two informal studies infer that children should learn to use various models to represent problem situations. Further, the "part-part-whole" method can help pupils learn to decide which operation is appropriate. It is felt that it is most effective to present these ideas over and over again for just a few minutes each week. While the research focuses on the elementary grades, the methods may also prove useful in some secondary situations. (MP)

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Teaching Children to Solve One-Step Word Problems TO THE EDUCATIONAL RESOURCES
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Helping children learn to solve problems has become a major goal for school mathematics (NCTM, 1980). An important first step for the development of these general problem-solving skills is to help children learn to solve one-step word problems. This is the stage at which children put "how to use an operation" knowledge together with "when to use an operation" knowledge as they are developing the four fundamental operations into useful tools in their problem-solving repertoire.

Unfortunately, many children are not very successful at deciding what operation is appropriate for solving a given word problem. Even teachers, who can easily solve these problems, have difficulty describing or explaining how to identify the key features of the problem that enable them to decide which of the four fundamental operations is appropriate.

Helping children learn how to decide when to use an operation for solving a problem is the focus for the remainder of this article. Two complementary approaches to this instructional problem are discussed. They involve (1) familiarizing students with various models for an operation and using them to represent problem situations, and (2) a more general approach relating the parts of a set to the whole set. In each case, a rationale for the approach is discussed and an action research project is described. The results of these informal studies are then used to make inferences about instructional strategies for teaching children to solve one-step word problems.

Modeling Word Problems

Even prior to formal instruction on addition and subtraction in grade one, children are highly successful in solving story problems (Carpenter, Hiebert, and Moser, 1979). Most children at this stage of development seem to directly model the action or relationships described in the problem (Carpenter, 1980).

As children learn more about the operations and models for them, they tend not to use the same variety of strategies they used at the early stage. Instead, they tend to begin categorizing different physical situations by the operation needed in order to solve the problem (Carpenter, 1980). For example, early in the year, first graders use counters differently when they solve take away, missing addend, and comparison situations. Eventually, these same children develop a more unified concept of subtraction and just recognize these situations as ones where subtraction is the appropriate operation to solve the problem. They no longer need to or do use counters to help them with the solution. In fact, children in the middle grades tend not

to think about the possibility of using concrete materials or even a diagram to aid them in the solutions of problems. Rather, they immediately try to identify the appropriate operation.

In order to be effective at this more advanced stage of development in solving word problems, children must develop a broad concept for the fundamental operations. They must recognize that an operation is appropriate for each of several different types of physical situations. For example, for any number of situations such as repeated additions, joining equivalent sets, arrays, or cartesian products, children must realize that multiplication is appropriate.

Developing this broad concept for an operation involves becoming familiar with various models for an operation and how they can be used to represent different situations. Children will have limited ability to solve word problems if they are familiar with only one model for an operation. For example, children who have only associated subtraction with the take away model will have difficulty knowing what operation to use for a comparison situation.

The premise that familiarization of models for an operation and how they can be used to represent different situations will help children learn to solve word problems is the basis for an action research project by Bushong (1974). Students were interviewed to determine the models they used in their thinking about multiplication and division. Instruction was provided for some models that the students were not using. A comparison of pretest and posttest scores on a word problem assessment was used to evaluate the effectiveness of the instruction.

Four fifth-grade students, who had been identified by their teachers as ineffective at solving word problems, were interviewed. It was found that they did not make use of the array as a model for multiplication or the partitive model for division. The thinking they did use was almost exclusively repeated addition and repeated subtraction, both thinking models closely related to the way they derived answers to multiplication and division problems.

A 16-item pretest confirmed that these students were not proficient at solving word problems where the appropriate model was an array or a partitive situation. They also performed poorly on measurement situations in division even though their thinking was generally repeated subtraction, which is closely related to a measurement model.

Five 20-minute lessons were provided for instruction on these three models. The first two introduced the array and then the use of arrays to model situations in multiplication word problems. The next two lessons introduced the partitive division model and the use of it to model situations in word problems. The last lesson was designed to help children use measurement models to represent division word problems.

The posttest was the same format as the pretest with similar problems using the same basic facts as the numbers involved. Each test included four items that corresponded to each of the models, repeated addition and arrays for multiplication, and partitive and measurement models for division. The results of both tests and the gains made on each are shown in Table 1.

TABLE 1
NUMBER AND PERCENT OF CORRECT RESPONSES
ON THE WORD PROBLEMS TEST

Model	Pretest	Posttest	Gain
Repeated Addition	11 (69%)	12 (75%)	1 (6%)
Array	7 (44%)	11 (69%)	4 (25%)
Partition	8 (50%)	13 (81%)	5 (31%)
Measurement	8 (50%)	11 (69%)	3 (19%)

The results clearly show that the gains made on the topics included during instruction were substantial. They ranged from a 19 percent gain for measurement division problems to a 31 percent gain on partitive division problems. Bushong indicates that the introduction of these models had a positive effect on the students' problem-solving skills. She also noted that the effect took place with very little instructional time. No more than 40 minutes of instruction was provided for any of the models.

The Part-Part-Whole Relationship

Many reasons have been given for the difficulties that children encounter while trying to solve word problems. Lack of reading skills has been suggested often. However, Knifong and Holton (1977) found that almost all of the students who could not solve a problem could correctly relate the problem to the researcher in their own words. Furthermore, they knew what was being asked. What many of them did not know was what operation was appropriate.

It is worth noting that several current textbooks have outlined a series of problem-solving steps for students. One of these steps involves deciding what operation is appropriate. However, little, if any, guidance is given students in helping them learn how to make this decision.

Wilson (1967) found that a part-part-whole treatment was very effective in helping fourth-grade children make decisions about whether to add or subtract to solve word problems. They were instructed to identify the wanted and the given in each problem. If the parts are given, add to find the whole. If the whole and a part is given, subtract to find the other part.

Brostrom (1979) used a similar technique for fifth graders, but she extended the scope of the study to include multiplication and division as well. The language "part-part-whole" was used for addition and subtraction problems. The corresponding thought process with the language "factor-factor-product" was used for multiplication and division problems.

There were eleven students in each of two treatments. The experimental treatment employed the part-part-whole decision-making process. The control treatment did the same assignments and received feedback on their work, but did not receive instruction about how to decide what operations to use. After each assignment the experimental treatment simply discussed each problem in terms of parts (factors) and wholes (products).

Each treatment had a fifteen-minute word problem lesson twice a week for six weeks. The first four lessons involved addition and subtraction, the next four multiplication and division, and the last four involved all four operations presented as mixed practice.

A 20-item pretest and posttest were given. There were five problems for each of the fundamental operations. Two points were possible for each problem, one for the correct process and one for the correct results.

Again the part-part-whole method proved to be very effective. The results are presented in Table 2. The results indicate the part-part-whole method for all four operations was as effective as Wilson found for addition and subtraction.

TABLE 2
MEAN SCORE AND PERCENT OF CORRECT
RESPONSES ON THE WORD PROBLEMS TEST

Treatment	Pretest	Posttest	Gain
experimental	29.0 (73%)	39.2 (98%)	10.2 (26%)
control	30.6 (77%)	33.6 (84%)	3.0 (8%)

Interviews were also conducted. Ten of the eleven students in the experimental treatment indicated an attempt to use the part-part-whole approach in deciding what operation to use for a word problem. The part-part-whole approach was apparently one that most students were not only able to use but also were willing to use. In contrast, eight of eleven students in the control treatment could not verbalize how they decided what operation to use. Two others said they would reread the problem to see if their answer made sense and one said that clue words were used. Brostrom referred to this as a hit-and-miss procedure. If their intuition did not help, they were unable to solve the problem.

An improvement in the attitudes of students toward word problems was also noted, particularly for those in the average and low average range. They felt confident in their application of the part-part-whole process and were delighted to have a method that took the guesswork out of solving word problems.

Essentially the same approach has successfully been employed in a first-grade classroom for addition and subtraction. The language of parts and wholes was introduced as addition and subtraction were introduced. Addends were simply the parts being joined to get the whole or the sum. Subtraction always involved using the whole and one part to determine the other part.

The children were orally presented two to four word problems about once a week from the time that addition was first introduced. As soon as subtraction had been introduced, problems involving both operations were presented in each session and part-whole discussions helped the children focus on how to decide what operation was appropriate.

As they became competent at solving word problems, they were given the opportunity to create their own problems to match a given addition or subtraction number sentence.

By the end of first grade, these students were capable of deciding what operation was appropriate by identifying parts and wholes. They also were capable of creating situations to match number sentences. The part-part-whole method had given them a firm base of problem-solving skills.

Commentary

The limited nature of the studies described indicate any generalizations from their results should be made cautiously. However, the fact that the results are consistent with other studies and that suggestions for instructional implementation have been successfully used indicates that some inferences about teaching can be made.

First, children should learn to use various models to represent problem situations. Most of the time that models are being used in the classroom, the focus is on using the model to derive an answer. Activities should be provided where the explicit purpose of the activity is to model the situation in the problem. Children can be asked to (1)

manipulate objects or (2) act out a situation. They can (3) draw a picture to show the problem. They can be asked to (4) match one of several models to a problem. They can (5) create a problem to match a model. In each case the emphasis should be on the modeling and not on the answer to the problem. Even older children who tend to skip over the model and try to relate the problem directly to an operation can profit from these experiences. The activities can be mainly oral and most will only take a few minutes of the mathematics lesson.

Second, the part-part-whole method can help children learn how to decide what operation is appropriate. They simply need to learn to identify the parts and the wholes. Then if they know the part, they add to find the whole. Or, if they know the whole and one part, they subtract to find the other part.

Rather than present all multiplication problems at the end of the multiplication chapter, as so many current textbooks do, this method demands the presentation of problems that require different operations; for example, some multiplication and some division. The main thrust should be to discuss parts (factors) and whole (products) and use that as a way to help decide what operation is appropriate. The problems can be presented orally or on a worksheet. Simply take a few minutes to discuss how the students know what operation to use.

Neither of these ideas can be learned in a lesson - or two. It is probably far more effective to present these ideas over and over again for just a few minutes each week. Two five-minute activities each week will have more impact than an equal amount of time devoted to the same topic but presented in a single lesson.

Solving word problems is an important goal for our students. There is much yet to be learned about how to help students learn these skills. But for the present, the activities suggested here offer realistic, easily implemented ideas which may help get your students on the right track.

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